Ceva's Theorem

Pallav Goyal

1. (Ceva's Theorem) Given ΔABC and points D, E and F on the lines BC, CE and AB respectively, the lines AD, BE and CF are concurrent if and only if

$$\frac{BD}{DC}.\frac{CE}{EA}.\frac{AF}{FB} = 1$$

2. Prove that, in any triangle, the three angle bisectors are concurrent, the three medians are concurrent and the three altitudes are concurrent.

3. Prove that the isogonal conjugates of three concurrent lines in a triangle are themselves concurrent.

4. (RMO 2012) Let ABC be a triangle. Let D and E be points on the line segment BC such that $BD = DE = EC$. Let F be the mid-point of AC. Let BF intersect AD in P and AE in Q respectively. Determine the ratio of the area of the triangle APQ to that of the quadrilateral $PDEQ$.

5. In ΔABC, AD, BE and C are concurrent lines AD is an altitude. Prove that AD bisects $\angle FDE$.

6. (APMO 1992) Suppose we are given a circle C with cetre O. A circle C' has centre X inside C and touches C at A. Another circle has centre Y inside C and touches C at B and touches C' at Z. Prove that the lines XB, YA and OZ are concurrent.

7. In ΔABC, AD, BE and CF are concurrent lines and P, Q and R are on EF, FD and DE respectively such that DP, EQ and FR are concurrent. Prove that AP, BQ and CR are also concurrent.

8. Let O denote an arbitrary point in a plane, M and N the feet of the perpendiculars dropped from O on the bisectors of the interior and exterior angle A of ΔABC; P and Q are defined in a similar manner for the angle B; R and T for the angle C. Prove that the lines MN, PQ and RT are concurrent or parallel.

9. (IMO 2016) ΔBCF has a right angle at B. Let A be the point on line CF such that $FA = FB$ and F lies between A and C. Point D is chosen such that $DA = DC$ and AC is the bisector $\angle DAB$. Point E is chosen such that $EA = ED$ and AD is the bisector of $\angle EAC$. Let M be the midpoint of CF. Let X be the point such that $AMXE$ is a parallelogram. Prove that BD, FX and ME are concurrent.